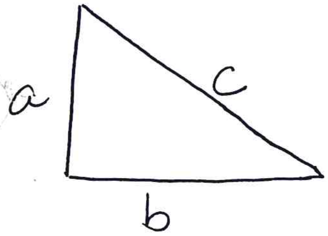
The Pythagorean Theorem



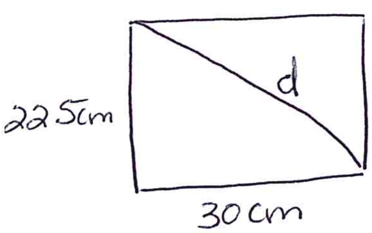
$$a^2 + b^2 = c^2$$

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

The hypotenuse is the longest side of a right triangle. It is the side opposite the 90° angle.

Find the hypotenuse

The size of a computer screen is actually the length of the diagonal of the screen. A computer screen measures 30cm by 22.5 cm. Determine the length of its diagonal.



d is the diagonal

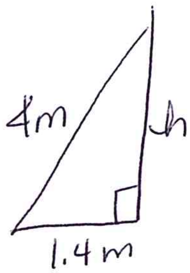
Use the Pythagorean theorem.

$$d^2 = a^2 + b^2$$
$$= (30)^2 + (22.5)^2$$
$$= 900 + 506.25$$
$$d^2 = 1406.25$$
$$\sqrt{d^2} = \sqrt{1406.25}$$
$$d = \pm 37.5 \qquad d = 37.5$$

don't use -37.5 b/c length is positive.

Find a shorter Side

Malka is changing a light bulb. She rests a 4m ladder against a vertical wall so the base is 1.4m from the wall. How high up the wall does the top of the ladder reach?



The ladder is the hypotenuse.

$a^2 + b^2 = c^2$

$(4)^2 = (1.4)^2 + b^2$

$16 = 1.96 + b^2$

$14.64 = b^2$

$3.7 \doteq b$ use only the positive root.

Find the area of a Right triangle

Calculate the area of the triangular sail on the sailboat.

$A = \frac{1}{2} bh$

Find a: $a^2 + b^2 = c^2$

$a^2 + (8)^2 = (11)^2$

$a^2 + 64 = 121$

$a^2 = 57$

$a \doteq 7.5$

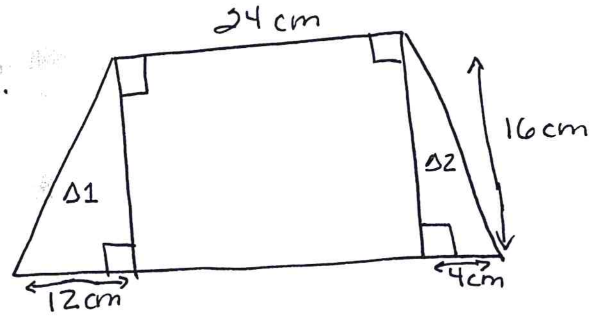
Now find area: $A = \frac{1}{2} bh$

$A = \frac{1}{2}(7.5)(8)$

$= 30 \text{ cm}^2$

Perimeter and Area of Composite Figures

Determine the area of the figure.
Determine the perimeter of the figure.



To find the area, find the area of the rectangle, and the 2 right triangles. Then add them together.

$$A_{rec} = l \cdot w$$
$$= (16)(24)$$
$$= 384 \text{ cm}^2$$

$$A_{\Delta 1} = \frac{1}{2} bh$$
$$= \frac{1}{2}(12)(16)$$
$$= 96 \text{ cm}^2$$

Total Area = $384 + 96 + 32 = 512 \text{ cm}^2$

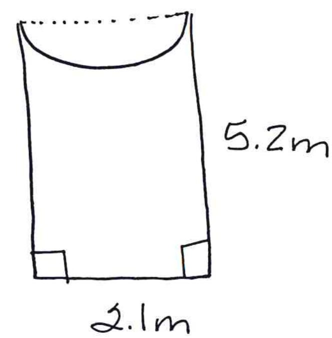
$$A_{\Delta 2} = \frac{1}{2} bh$$
$$= \frac{1}{2}(4)(16)$$
$$= 32 \text{ cm}^2$$

or use Area of trapezoid formula

$$A_{Trap} = \frac{1}{2}(b_1 + b_2)h$$
$$= \frac{1}{2}(24 + 40)(16)$$
$$A_{Trap} = \frac{(64)(16)}{2} = \frac{1024}{2} = 512 \text{ cm}^2$$

Area of a Composite figure, by subtraction and Perimeter

A hotel has a walkway leading to a semicircular fountain. Find the area. The walkway will have a border in colored tile. Find the perimeter.



Area

The walkway is a rectangle with a semicircle cut out of it.

Find the area of the rectangle minus the circle.

$A_r = l \times w$

$= (5.2)(2.1)$

$= 10.92$

a semicircle is $\frac{1}{2}$ a circle. The area is $\frac{1}{2}$ a circle's area. $\frac{1}{2}\pi r^2$

$A_{sc} = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi (1.05)^2$ $\qquad r = \frac{2.1}{2} = 1.05$

$= 1.73$

$A_w = A_r - A_{sc}$

$= 10.92 - 1.73$

$= 9.19$

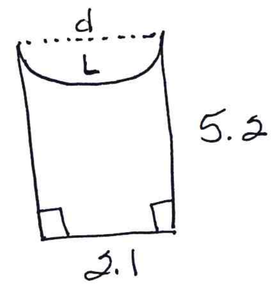
$\doteq 9.2\ m^2$

The perimeter of the walkway consists of three sides of the rectangle and the semicircular arc.

→ find the length of the arc:

the circumference of a circle is $C = \pi d$, so the length of a semicircular arc is half the circumference.

$$L = \frac{1}{2}\pi d$$



$$L = \frac{1}{2}\pi d$$
$$= \frac{1}{2}\pi(2.1)$$
$$\doteq 3.3$$

Perimeter is the distance around the outside of the walkway.

$$P = 3.3 + 5.2 + 2.1 + 5.2$$
$$= 15.8 \text{ m}$$

A composite figure is made up of more than one simple shape.

To find the area, add or subtract areas from a shape.

To find the perimeter, add the distances around the outside of the figure.

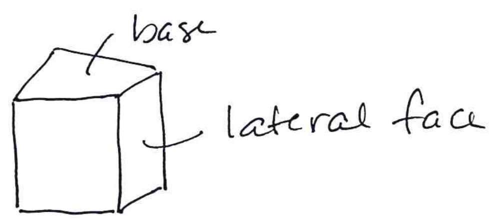
Surface Area and Volume of Prisms and Pyramids

Surface Area - the number of square units needed to cover the surface of a 3-dimensional object.

Volume - the amount of space that an object occupies, measured in cubic units.

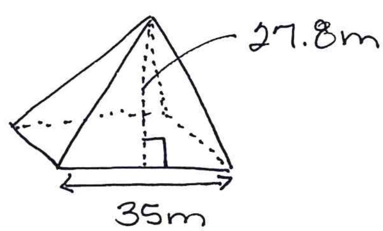
Pyramid - a polyhedron whose base is a polygon and whose faces are triangles that meet at a common vertex.

lateral faces - the faces of a prism or pyramid that are not bases.



Surface area of a pyramid

Calculate the surface area of the square-based pyramid.



$A_{base} = 35 \times 35 = 1225$

$P_{base} = 35 + 35 + 35 + 35 = 140$

$$SA_{pyr} = Area_{base} + \frac{1}{2}(P_{base} \cdot h)$$

$$= 1225 + \frac{1}{2}(140 \cdot 27.8)$$

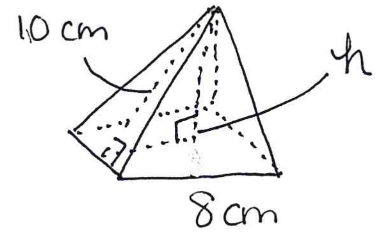
$$= 1225 + \frac{1}{2}(3892)$$

$$= 1225 + 1946$$

$$= 3171 \text{ m}^2$$

Volume of a pyramid

$V_{pyramid} = \frac{1}{3}(A_b)(h)$



Find the volume of the pyramid. Express in liters.

first find the height, h, using the Pythagorean theorem.

$a^2 + b^2 = c^2$ in our pyramid: $a = h$

$b = 4 \quad (8 \div 2 = 4)$

$c = 10$

$h^2 + 4^2 = 10^2$ solve for h

$h^2 = 10^2 - 4^2$

$h^2 = 100 - 16$

$h^2 = 84$

$h = \sqrt{84} \doteq 9.2$

Now find the volume

$V_{pyr} = \frac{1}{3}(A_b)(h)$ $\qquad A_b = (8)(8) = 64$

$= \frac{1}{3}(64)(9.2)$

$\doteq 196 \text{ cm}^3$

$1 \text{ cm}^3 = 1 \text{ mL}$

$1000 \text{ mL} = 1 \text{ L}$

$196 \text{ cm}^3 = 196 \text{ mL}$

$196 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}} = .196 \text{ L}$

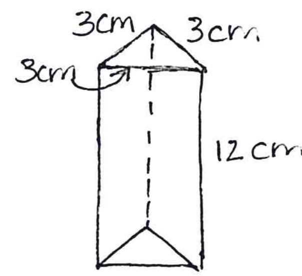
Surface Area and Volume of a Triangular Prism

$V_{\Delta prism} = A_b \cdot h$ $SA_{\Delta prism}$ = sum of the areas of the bases and faces

Chocolate is sometimes packaged in a box that is shaped like a triangular prism.

Calculate the amount of material required to make this box.

Calculate the volume of this box.



The amount of material required is the surface area. This is 2 triangles (the top and bottom) and 3 congruent rectangles (the faces)

Find the height of each triangle

Use Pythagorean theorem

$a^2 + b^2 = c^2$

$h^2 + (1.5)^2 = (3)^2$

$h^2 + 2.25 = 9$

$h^2 = 9 - 2.25$

$h^2 = 6.75$

$h = \sqrt{6.75}$

$h \doteq 2.6$

Now calculate the surface area.

$SA = \Sigma A_{bases} + \Sigma A_{faces}$ Σ means sum of

$SA_{\Delta prism} = (3.9 + 3.9) + (36 + 36 + 36)$

$= 7.8 + 108$

$= 115.8$

$\doteq 116\ cm^2$

needed to build the box

$A_b = \frac{1}{2} bh$

$= \frac{1}{2}(3)(2.6)$

$= 3.9$

$A_f = l \cdot w$

$= (12)(3)$

$= 36$

Now find the volume

$V_{\Delta prism} = A_b \cdot h$

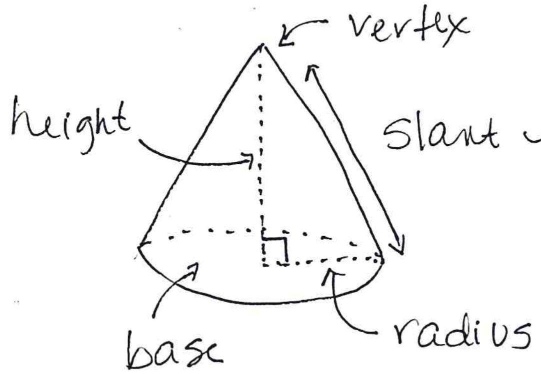
$= (3.9)(12)$

$= 46.8$

$\doteq 47\ cm^3$

Surface Area of a Cone

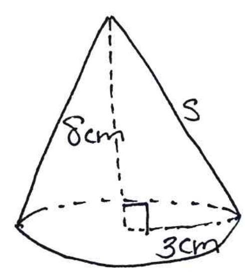
Cone - a 3-dimensional figure with a circular base and a curved lateral surface that extends from the base to a point called the vertex



$SA_{cone} = A_b + (\frac{1}{2} C * \text{height of slant})$

$= \pi r^2 + \pi r s$

Find the surface area of the cone



determine s using the Pythagorean theorem

$s^2 = h^2 + r^2$

$s^2 = 8^2 + 3^2$

$s^2 = 64 + 9$

$s^2 = 73$

$s = \sqrt{73}$

$s \doteq 8.5$

$SA_{cone} = \pi r^2 + \pi r s$

$= \pi (3)^2 + \pi (3)(8.5)$

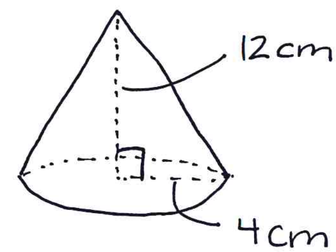
$\doteq 108 \text{ cm}^2$

Volume of a Cone

$V_{cone} = \frac{1}{3} Bh$ $\qquad A_b = B$ = area of base

$= \frac{1}{3} A_b \cdot h$

Determine the volume of the cone.



$V_{cone} = \frac{1}{3} A_b \cdot h$

$A_b = \pi r^2$ – the base is a circle

$= \pi (4)^2$

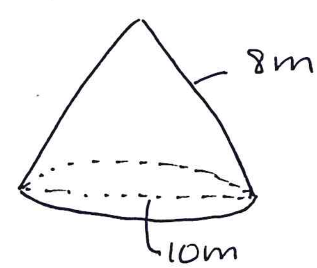
$= 50.27$

$V_{cone} = \frac{1}{3} (50.27)(12)$

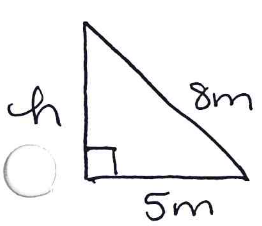
$= 201.08$

$\doteq 201 \text{ cm}^3$

A pile of sand has a base diameter of 10 m and a slant height of 8 m. Determine the volume of sand in the pile to the nearest meter.^{3}



First find the height of the cone.



you know the radius is 5m because the diameter is 10m

Use the Pythagorean Theorom

$5^2 = h^2 + r^2$

$8^2 = h^2 + 5^2$

$64 - 25 = h^2$

$h^2 = 39$

$h = \sqrt{39}$

$h \doteq 6.2$

Now find the Volume

$V_{cone} = \frac{1}{3} A_b \cdot h$

$A_b = \pi r^2 = \pi \cdot 5^2$

$= \frac{1}{3} (\pi \cdot 5^2)(6.2)$

$\doteq 162 \text{ m}^3$

Find the Height of a cone

A conical container has a volume of 210 cm^3. Its diameter is 8 cm. Find the height of the container.

$V_{cone} = \frac{1}{3} \pi r^2 h$

$210 = \frac{1}{3} \pi (4)^2 h$

$210 = \frac{16\pi}{3} h$

$3(210) = 16\pi h$

$h = \frac{3(210)}{16\pi}$

$h = 12.5 \text{ cm}$

Surface Area of a Sphere

Sphere: a round ball-shaped object
a set of points in space that are a given distance (radius) from a fixed point (centre).

$SA_{sphere} = 4\pi r^2$

The average diameter of the eyeball is about 2.5 cm. Calculate the surface area to the nearest tenth of a square centimetre.

$d = 2.5 \qquad r = \dfrac{2.5}{2} = 1.25 \text{ cm}$

$SA_{sphere} = 4\pi r^2$
$= 4\pi (1.25)^2$
$\doteq 19.6 \text{ cm}^2$

Find the radius of a baseball that has a surface area of 215 cm^2? Round to the nearest tenth of a centimetre.

$SA_{sphere} = 4\pi r^2$

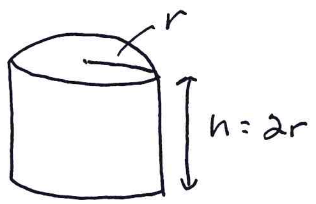
$215 = 4\pi r^2 \qquad r \doteq 4.1$

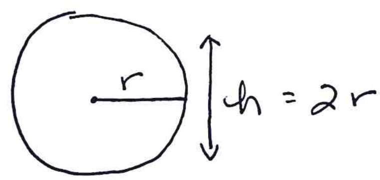
$\dfrac{215}{4\pi} = r^2$

$\sqrt{17.1} = r^2$

Volume of a Sphere

The volume of a shpere is $\frac{2}{3}$ the volume of a cylinder with the same radius and a height equal to the diameter of the sphere. If the sphere has radius, r, then the cylinder has a base radius, r, and a height 2r.





$$V_{sphere} = \frac{4}{3}\pi r^3$$

A spherical stone just fits inside a plastic cube with edges 10 cm.

Find the volume of the stone.

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(5)^3$$

$$\doteq 524 \text{ cm}^3$$

How much empty space is left in the cube?

Find the volume of the cube

$V_{cube} = s^3$

$= (10)^3$

$= 1000$

$V_{empty\ space} = V_{cube} - V_{sphere}$

$= 1600 - 524$

$= 476\ cm^3$ of empty space.