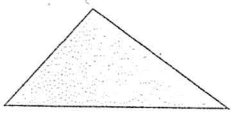
Geometric Relationships - Review of Terms

Classify Triangles

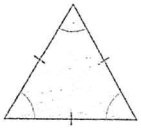
Triangles can be classified using their side lengths or their angle measures.

scalene triangle
- no equal sides
- no equal angles

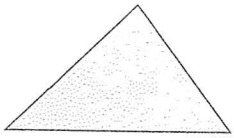


isosceles triangle
- two equal sides
- two equal angles

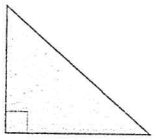
equilateral triangle
- three equal sides
- three equal angles



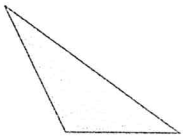
acute triangle
- three acute angles (less than 90°)



right triangle
- one right angle (90°)



obtuse triangle
- one obtuse angle (between 90° and 180°)



Classify Polygons

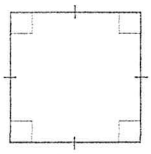
A **polygon** is a closed figure formed by three or more line segments.

A **regular polygon** has all sides equal and all angles equal.

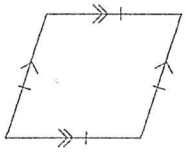
Some quadrilaterals have special names. A regular quadrilateral is a **square**. An irregular quadrilateral may be a **rectangle**, a **rhombus**, a **parallelogram**, or a **trapezoid**.

Number of Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon

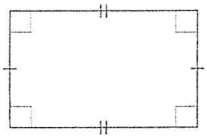
square



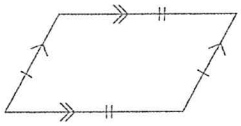
rhombus



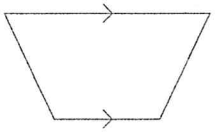
rectangle



parallelogram

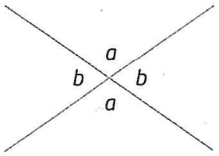


trapezoid

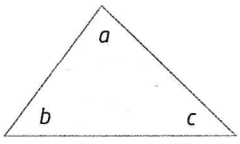


Angle Properties

When two lines intersect, the **opposite angles** are equal.



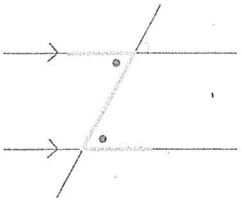
The sum of the interior angles of a triangle is 180°.



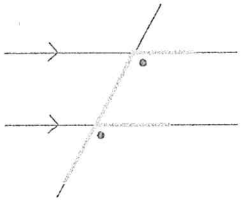
$a + b + c = 180°$

When a transversal crosses parallel lines, many pairs of angles are related.

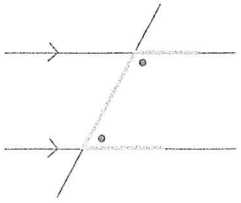
alternate angles are equal



corresponding angles are equal



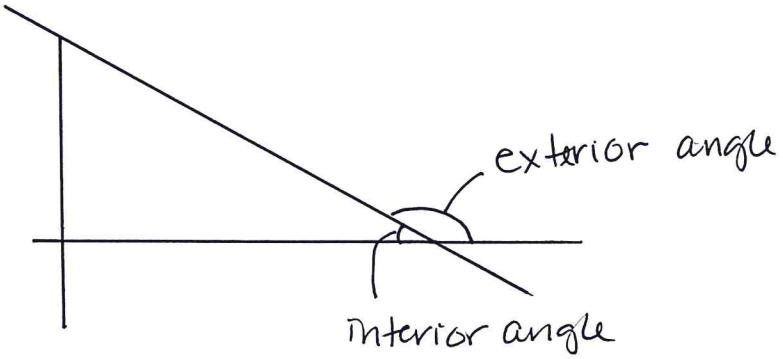
co-interior angles have a sum of 180°

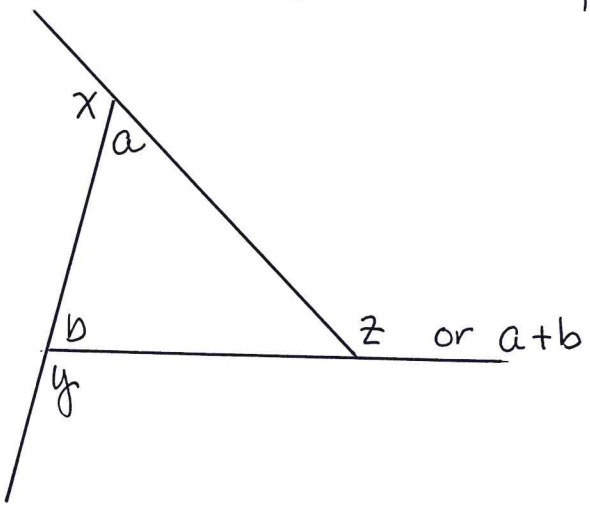


Vertex - the point where two or more sides meet.

Interior angle - angle formed on the inside of a polygon by two sides meeting at a vertex.

Exterior angle - angle formed on the outside of a geometric shape by extending one of the sides past a vertex.



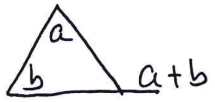


$x+y+z=360°$

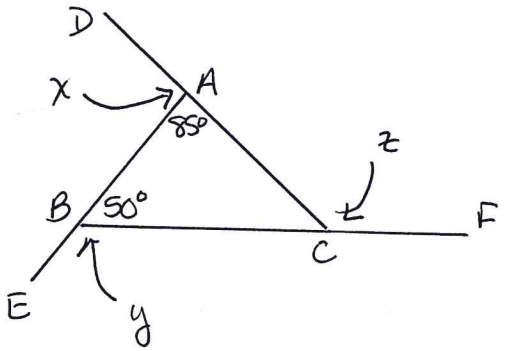
The sum of the exterior angles of a triangle is 360°

$$x+y+z=360°$$

The exterior angle at each vertex of a triangle is equal to the sum of the interior angles at the other two vertices.



Find the measures of the exterior angles of $\triangle ABC$.



At vertex A and vertex B, the interior and exterior angles together form a 180° angle.

$\measuredangle DAB + \measuredangle CAB = 180°$ We know $\measuredangle CAB = 85°$
We need to solve for $\measuredangle DAB$

$\measuredangle DAB = 180° - \measuredangle CAB$

$\measuredangle DAB = 180° - 85°$

$x = \measuredangle DAB = 95°$

$\measuredangle EBC + \measuredangle ABC = 180°$ We know $\measuredangle ABC = 50°$
We need to solve for $\measuredangle EBC$

$\measuredangle EBC = 180° - \measuredangle ABC$

$\measuredangle EBC = 180° - 50°$

$y = \measuredangle = EBC = 130°$

To find $\measuredangle ACF$:

The exterior angle at a vertex of a triangle is equal to the sum of the interior angles at the other two vertices.

$\measuredangle ACF = \measuredangle CAB + \measuredangle ABC$

$= 85° + 50°$

$= 135°$

Or

The sum of the exterior angles of a triangle is 360°.

$\measuredangle ACF + \measuredangle DAB + \measuredangle EBC = 360°$

$\measuredangle ACF = 360° - \measuredangle DAB - \measuredangle EBC$

$\measuredangle ACF = 360° - 95° - 130°$

$z = \measuredangle ACF = 135°$

So the exterior angle measurements are.

$x = \measuredangle DAB = 95°$

$y = \measuredangle EBC = 130°$

$z = \measuredangle ACF = 135°$

An equilateral triangle is also equiangular.

Equiangular is having all angles equal.

The sum of the interior angles of any triangle is 180°. If all the angles are equal then each one must be $\frac{180°}{3} = 60°$

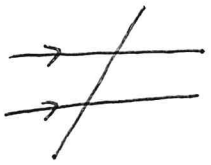
Since the interior angles are equal the exterior angles must be equal. The sum of the exterior angles of any triangle is 360°. If they are equal the each one must be $\frac{360°}{3} = 120°$.

Angle Relationships

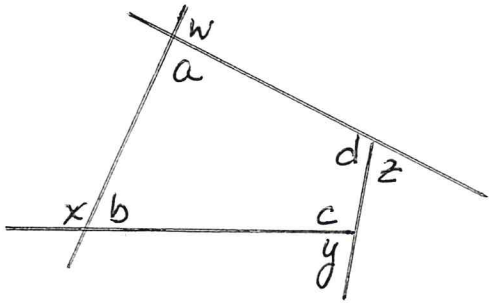
Adjacent - adjoining or next to

Supplementary - adding to 180°

Transversal - line intersecting two or more lines



Angle measurements of a quadrilateral



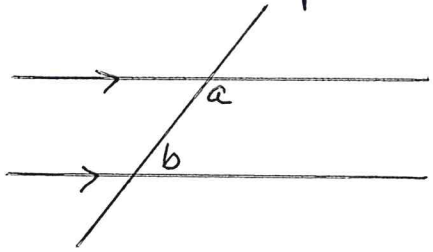
The sum of the interior angles of a quadritateral is 360°.

The sum of the exterior angles of a quadrilateral is 360°.

Interior angles : $a+b+c+d = 360°$

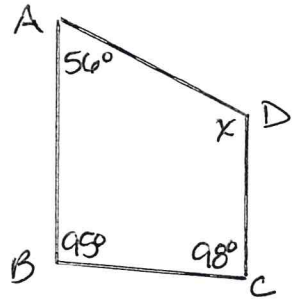
Exterior angles : $w+x+y+z = 360°$

Where a transversal crosses two parallel lines, angles that form a "C" pattern are supplementary.



$a+b = 180°$

Find the unknown angle measurements



The sum of the interior angles of a quadrilateral is 360°.

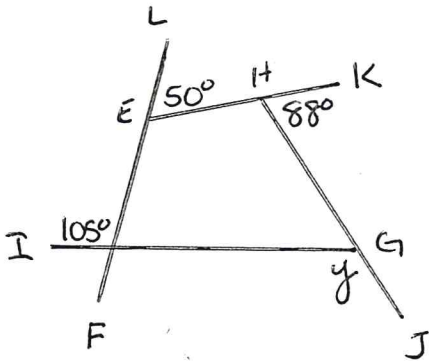
So:

$\angle A + \angle B + \angle C + \angle D = 360°$

$56° + 95° + 98° + x = 360°$

$249° + x = 360°$

$x = 111°$



The sum of the exterior angles of a quadrilateral is 360°.

So:

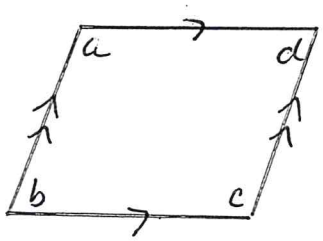
$\angle EFI + \angle LEH + \angle KHG + \angle JGF = 360°$

$105° + 50° + 88° + y = 360°$

$243° + y = 360°$

$y = 117°$

Angle properties of a parallelogram



adjacent angles are supplementary

$a + b = 180°$

$c + d = 180°$

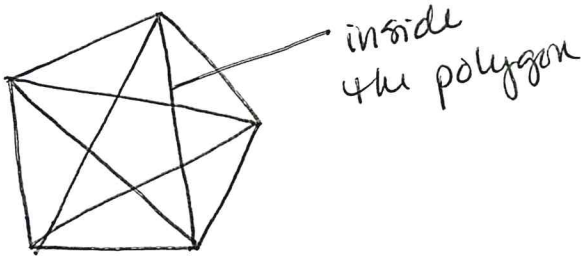
opposite angles in a parallelogram are equal

$a = c$

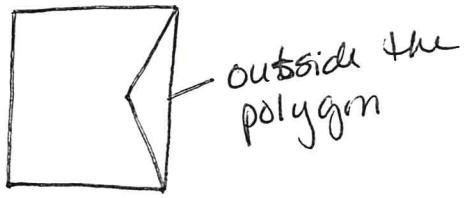
$b = d$

Polygons

Convex polygon - a polygon with no part of any line segment joining two points on the polygon outside the polygon.



Concave polygon - a polygon with parts of some line segments joining two points on the polygon outside the polygon.



polygons are named by the number of sides it has

3 sides - triangle

4 sides - quadrilateral

5 sides - pentagon

6 sides - hexagon

The sum of the exterior angles of a convex polygon is 360°.

For a polygon with "n" sides, the sum of the interior angles is $180(n-2)$.

Find the sum of the interior angles of an octagon.

sum of interior angles $= 180(n-2)$

for an octagon $n = 8$ because it has 8 sides

$$\begin{aligned} \text{sum of interior angles} &= 180(8-2) \\ &= 180(6) \\ &= 1080^\circ \end{aligned}$$

How many sides does a polygon have if each of its interior angles measures 140°?

An n-sided polygon has n interior angles. If each interior angle measures 140°, their sum, in degrees, is $140n$. The sum of the interior angles of any polygon is $180(n-2)$.

So:
$$\begin{aligned} 180(n-2) &= 140n \\ 180n - 360 &= 140n \\ 40n - 360 &= 0 \\ 40n &= 360 \\ n &= 9 \end{aligned}$$

So: A polygon with interior angles of 140° has 9 sides.