I. Model Problems

To simplify fractional exponents, rewrite the expression as a radical raised to a power. The denominator of the fractional exponent is the root and the numerator is the power.

In other words:
$$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^n$$

Example 1 Write $27^{2/3}$ as a radical and simplify.

$$27^{2/3} = (\sqrt[3]{27})^2$$
$$= 3^2$$

Rewrite as a radical.

$$=3^{2}$$

Simplify.

Simplify.

The answer is 9.

Sometimes you need to write a radical expression using a fractional exponent.

Example 2 Write $\sqrt[4]{y^6}$ using a fractional exponent.

$$\sqrt[4]{y^6} = y^{6/4}$$

Rewrite as a fractional exponent.

$$= v^{3/2}$$

Simplify.

The answer is $y^{3/2}$.

Sometimes there will be many variables in the radicand. Simplify each variable one at a time, then multiply.

Example 3 Simplify $\sqrt[3]{8x^5y^6z^{11}}$.

$$\sqrt[3]{8x^5y^6z^{11}} = \sqrt[3]{8} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{z^{11}}$$
 Rewrite the expression.

$$= 2 \cdot x \sqrt[3]{x^2} \cdot y^2 \cdot z^3 \sqrt[3]{z^2}$$
$$= 2xy^2 z^3 \cdot \sqrt[3]{x^2 z^2}$$

Simplify.

$$=2xy^2z^3\cdot\sqrt[3]{x^2z^2}$$

Simplify.

The answer is $2xy^2z^3 \cdot \sqrt[3]{x^2z^2}$.

Practice #2 Radicals and Rational Exponents

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Write each expression in exponential form.

1)
$$(\sqrt{7m})^5$$

2)
$$(\sqrt[4]{r})^7$$

3)
$$(\sqrt[3]{6x})^4$$

4)
$$\frac{1}{\sqrt{6n}}$$

Simplify.

5)
$$(b^4)^{-\frac{3}{2}}$$

6)
$$(v^9)^{\frac{5}{3}}$$

7)
$$(x^{12})^{-\frac{2}{3}}$$

8)
$$(125n^3)^{-\frac{5}{3}}$$

Write each expression in exponential form.

9)
$$(\sqrt{5k})^5$$

10)
$$\sqrt[3]{4a}$$

11)
$$\frac{1}{(\sqrt[3]{4p})^2}$$

12)
$$\sqrt{x}$$

Write each expression in radical form.

13)
$$(3n)^{\frac{1}{4}}$$

14)
$$m^{\frac{1}{4}}$$

15)
$$(2x)^{\frac{5}{6}}$$

16)
$$r^{\frac{5}{2}}$$

Simplify. Answers should contain only positive exponents.

17)
$$\frac{\left(y^{\frac{1}{4}}\right)^{2} \cdot x^{-\frac{1}{2}}y^{2}}{x^{\frac{7}{4}}y^{\frac{3}{2}}}$$

$$18) \frac{nm^{\frac{7}{4}}}{\left(mn^{\frac{5}{4}}\right)^2 \cdot mn^{-1}}$$

19)
$$\left(\frac{a^{\frac{5}{3}}b^2 \cdot a^{\frac{5}{4}}b^{\frac{3}{2}}}{(a^2)^{-4}}\right)^3$$

$$20) \left(\frac{x^{-1}y^{\frac{1}{4}}}{x^{\frac{2}{3}}y^{-1} \cdot x^{\frac{5}{4}}y^{\frac{5}{3}}} \right)^{\frac{5}{4}}$$

Simplify.

$$(1)(n^4)^{\frac{3}{2}}$$

(2)
$$(27p^6)^{\frac{2}{3}}$$

$$(3)(25b^6)^{-\frac{3}{2}}$$

$$(4)$$
 $(64m^4)^{\frac{3}{2}}$

5)
$$(a^8)^{\frac{3}{2}}$$

6)
$$(9r^4)^{\frac{1}{2}}$$

7)
$$(81x^{12})^{\frac{5}{4}}$$

8)
$$(216r^9)^{\frac{1}{3}}$$

Simplify. Your answer should contain only positive exponents.

9)
$$2m^2 \cdot 4m^{\frac{3}{2}} \cdot 4m^{-2}$$

10)
$$3b^{\frac{1}{2}} \cdot b^{\frac{4}{3}}$$

$$11) \left(p^{\frac{3}{2}}\right)^{-2}$$

$$12) \left(a^{\frac{1}{2}}\right)^{\frac{3}{2}}$$

$$\underbrace{13}_{4x^{\frac{4}{3}}}^{-\frac{7}{4}}$$

14)
$$\frac{4x^2}{2x^2}$$

$$(15)) \frac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}}y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$$

$$16) \frac{3y^{\frac{1}{4}}}{4x^{-\frac{2}{3}}y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$$

$$17) \left(m \cdot m^{-2} n^{\frac{5}{3}}\right)^2$$

$$(18) \left(a^{-1}b^{\frac{1}{3}} \cdot a^{-\frac{4}{3}}b^2 \right)^2$$

$$\underbrace{(19)}_{x^{2}y^{-2}} \left(\frac{x^{\frac{1}{2}}y^{-2}}{yx^{\frac{-7}{4}}} \right)^{4}$$

$$\underbrace{(x^{3}y^{2})^{\frac{3}{2}}}_{(x^{-1}y^{-\frac{2}{3}})^{\frac{1}{4}}}$$

$$21) \frac{\left(x^{-\frac{1}{2}}y^{2}\right)^{-\frac{5}{4}}}{x^{2}y^{\frac{1}{2}}}$$

22)
$$\frac{\left(x^{-\frac{1}{2}}y^{4}\right)^{\frac{1}{4}}}{\frac{2}{x^{3}}y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}}y^{\frac{1}{2}}}$$