I. Model Problems

To simplify fractional exponents, rewrite the expression as a radical raised to a power. The denominator of the fractional exponent is the root and the numerator is the power.

In other words: $x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Example 1 Write $27^{2/3}$ as a radical and simplify.

$27^{2/3} = \left(\sqrt[3]{27}\right)^2$ Rewrite as a radical.

$= 3^2$ Simplify.

$= 9$ Simplify.

The answer is **9.**

Sometimes you need to write a radical expression using a fractional exponent.

Example 2 Write $\sqrt[4]{y^6}$ using a fractional exponent.

$\sqrt[4]{y^6} = y^{6/4}$ Rewrite as a fractional exponent.

$= y^{3/2}$ Simplify.

The answer is **$y^{3/2}$.**

Sometimes there will be many variables in the radicand. Simplify each variable one at a time, then multiply.

Example 3 Simplify $\sqrt[3]{8x^5 y^6 z^{11}}$.

$\sqrt[3]{8x^5 y^6 z^{11}} = \sqrt[3]{8} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{z^{11}}$ Rewrite the expression.

$= 2 \cdot x\sqrt[3]{x^2} \cdot y^2 \cdot z^3\sqrt[3]{z^2}$ Simplify.

$= 2xy^2z^3 \cdot \sqrt[3]{x^2z^2}$ Simplify.

The answer is $2xy^2z^3 \cdot \sqrt[3]{x^2z^2}$.

Practice #2 Radicals and Rational Exponents

Write each expression in exponential form.

1) $\left(\sqrt{7m}\right)^5$

2) $\left(\sqrt[4]{r}\right)^7$

3) $\left(\sqrt[3]{6x}\right)^4$

4) $\dfrac{1}{\sqrt{6n}}$

Simplify.

5) $\left(b^4\right)^{-\frac{3}{2}}$

6) $\left(v^9\right)^{\frac{5}{3}}$

7) $\left(x^{12}\right)^{-\frac{2}{3}}$

8) $\left(125n^3\right)^{-\frac{5}{3}}$

Write each expression in exponential form.

9) $\left(\sqrt{5k}\right)^5$

10) $\sqrt[3]{4a}$

11) $\dfrac{1}{\left(\sqrt[3]{4p}\right)^2}$

12) $\sqrt{x}$

Write each expression in radical form.

13) $(3n)^{\frac{1}{4}}$

14) $m^{\frac{1}{4}}$

15) $(2x)^{\frac{5}{6}}$

16) $r^{\frac{5}{2}}$

Simplify. Answers should contain only positive exponents.

17) $\dfrac{\left(y^{\frac{1}{4}}\right)^2 \cdot x^{-\frac{1}{2}} y^2}{x^{\frac{7}{4}} y^{\frac{3}{2}}}$

18) $\dfrac{nm^{\frac{7}{4}}}{\left(mn^{\frac{5}{4}}\right)^2 \cdot mn^{-1}}$

19) $\left(\dfrac{a^{\frac{5}{3}} b^2 \cdot a^{\frac{5}{4}} b^{\frac{3}{2}}}{\left(a^2\right)^{-4}}\right)^3$

20) $\left(\dfrac{x^{-1} y^{\frac{1}{4}}}{x^{\frac{2}{3}} y^{-1} \cdot x^{\frac{5}{4}} y^{\frac{5}{3}}}\right)^{\frac{5}{4}}$