Functions And Equivalent Algebraic Expressions

Determine if two functions are Equivalent:

- test with different values of x
- simplify expressions
- graph on calculator

Rational expression! the quotient of 2 polynomials, $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$

$$f(x) = \frac{x^2-2x-8}{x^2-x-12} \qquad g(x) = \frac{x+2}{x+3}$$

x	$\frac{x^2-2x-8}{x^2-x-12}$	$\frac{x+2}{x+3}$
-1	$\frac{(-1)^2-2(-1)-8}{(-1)^2-(-1)-12}$ $\frac{1+2-8}{1+1-12}$ $\frac{1}{2}$	$\frac{-1+2}{-1+3}$ $\frac{1}{2}$
0	$\frac{0^2-2(0)-8}{0^2-0-12}$ $\frac{-8}{-12}$ $\frac{2}{3}$	$\frac{0+2}{0+3}$ $\frac{2}{3}$
1	$\frac{1^2-2(1)-8}{1^2-(1)-12}$ $\frac{-9}{-12}$ $\frac{3}{4}$	$\frac{1+2}{1+3}$ $\frac{3}{4}$

∴ same values appear to be equivalent

Simplify

$g(x) = \frac{x+2}{x+3}$ is simplified

$$f(x) = \frac{x^2-2x-8}{x^2-x-12}$$

factor numerator + denominator

$$= \frac{(x-4)(x+2)}{(x-4)(x+3)}$$

cancel common factor

$$= \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)}$$

$$= \frac{(x+2)}{(x+3)}$$: appears to be equivalent

Graph

Plot them 1 at a time

$\frac{x^2-2x-8}{x^2-x-12}$ has a break in the graph

Look a Table Error at $x=4$

since denominator cannot be zero $x \neq -3$ or $x \neq 4$

Determine Restrictions

Simplify expressions and determine any restrictions

$$\frac{6x^2-7x-5}{3x^2+x-10}$$

factor

$6x^2+3x-10x-5$

$3x(2x+1)-5(2x+1)$

$(3x-5)(2x+1)$

$$\frac{(2x+1)(3x-5)}{(3x-5)(x+2)}$$

$3x^2+6x-5x-10$

$(3x-5)(x+2)$

find your restrictions

$3x-5=0$

$3x=5$

$x=\frac{5}{3}$

$x+2=0$

$x=-2$

so $x \neq \frac{5}{3}$ or $x \neq -2$

simplify by dividing common factors

$$\frac{(2x+1)\cancel{(3x-5)}}{\cancel{(3x-5)}(x+2)}$$

$$\frac{2x+1}{x+2}$$

so $\dfrac{6x^2-7x-5}{3x^2+x-10} = \dfrac{2x+1}{x+2}$, $x \neq -2$, $x \neq 5/3$

Operations with Rational Expressions 2.2

Rational Expression: the quotient of two polynomials,

$\frac{p(x)}{q(x)}$, where $q(x) \neq 0$

Simplify and State Restrictions

$$\frac{4x^2}{3x} \cdot \frac{12x^3}{2x} = \frac{48x^5}{6x^2} = 8x^3$$

Restrictions : Remember denominator cannot be 0.

$x \neq 0$

so $\frac{4x^2}{3x} \cdot \frac{12x^3}{2x} = 8x^3$, $x \neq 0$

$$\frac{10ab^2}{4a} \div \frac{15a^2}{12b^2}$$

$$\frac{10ab^2}{4a} \cdot \frac{12b^2}{15a^2} = \frac{120ab^4}{60a^3} = \frac{2b^4}{a^2}$$

Remember: original expression $a + b$ are both in denominator so $a \neq 0$ $b \neq 0$

$$\frac{2x^2-8x}{x^2-3x-10} \div \frac{4x^2}{x^2-9x+20}$$

$$\frac{2x^2-8x}{x^2-3x-10} \cdot \frac{x^2-9x+20}{4x^2}$$

invert and multiply

factor

$$\frac{2x(x-4)}{(x-5)(x+2)} \cdot \frac{(x-5)(x-4)}{4x^2}$$

cancel common factors

$$\frac{2\cancel{x}(x-4)}{(x\cancel{-5})(x+2)} \cdot \frac{(x\cancel{-5})(x-4)}{\cancel{4x^2}\ 2x}$$

multiply

$$\frac{(x-4)^2}{2x(x+2)}$$

State restrictions: must include all times when denominator can be zero

original expression: $(x-5)$ $\quad x-5=0 \quad x=5$

$(x+2)$ $\quad x+2=0 \quad x=-2$

$(x-4)$ $\quad x-4=0 \quad x=4$

when inverted : $4x^2$ $\quad x=0$

restrictions: $x \neq 5$, $x \neq -2$, $x \neq 4$, $x \neq 0$

$$\frac{1}{5x} + \frac{1}{2x}$$

Find the LCM of the denominator

$5x + 2x \quad : \quad 5 \cdot 2 \cdot x = 10x$

this gives you the LCD

least common denominator

$\frac{1}{5x} = \frac{?}{10x}$

Multiply top and bottom by same number to give $10x$

$\frac{1}{2x} = \frac{?}{10x}$

$\frac{1}{5x}\frac{(2)}{(2)} = \frac{2}{10x} \qquad \frac{1}{2x}\frac{(5)}{(5)} = \frac{5}{10x}$

Now add the numbers

$\frac{2}{10x} + \frac{5}{10x} = \frac{7}{10x}$

Restrictions: $x \neq 0$

$$\frac{x+9}{x^2+2x-48} - \frac{x-9}{x^2-x-30}$$

factor denominator to find LCM

$x^2+2x-48$ $\quad$ x^2-x-30

$(x+8)(x-6)$ $\quad$ $(x-6)(x+5)$

LCD: $(x+8)(x-6)(x+5)$

$$\frac{x+9}{x^2+2x-48} = \frac{(x+9)(x+5)}{(x+8)(x-6)(x+5)}$$

multiply each expression by a fraction $=1$ to give LCD

$$\frac{x-9}{x^2-x-30} = \frac{(x-9)(x+8)}{(x+8)(x-6)(x+5)}$$

$$\frac{(x+9)(x+5)}{(x+8)(x-6)(x+5)} - \frac{(x-9)(x+8)}{(x+8)(x-6)(x+5)}$$

multiply the numerators

$$\frac{x^2+14x+45}{(x+8)(x-6)(x+5)} - \frac{x^2-x-72}{(x+8)(x-6)(x+5)}$$

Add numerators (remember to change the signs for the 2nd expression)

$$\frac{x^2+14x+45-x^2+x+72}{(x+8)(x-6)(x+5)}$$

$$\frac{15x+117}{(x+8)(x-6)(x+5)}$$

restrictions:

$x \neq -8$ $\quad$ $x \neq -5$

$x \neq 6$

Horizontal And Vertical Translations

Transformations that shift the function up, down, left or right without affecting the shape.

$g(x) = f(x) + c$ vertical translation

"only the y changes"

if c is positive : move up c units

c is negative : move down c units

$g(x) = f(x-d)$ horizontal translation

"only the x changes"

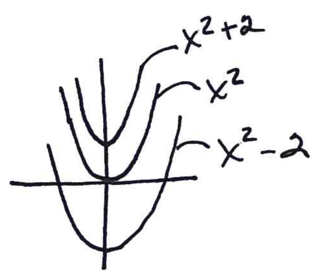
if d is positive : move right d units

d is negative : move left d units

On calculator

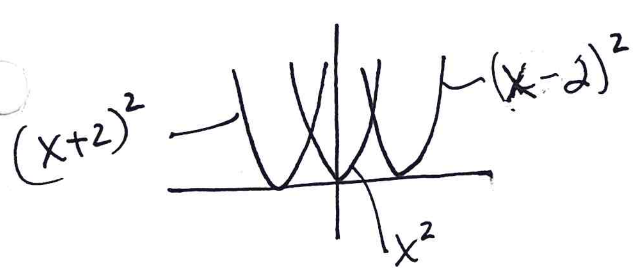
graph $f(x) = x^2$, $g(x) = x^2 + 2$, $h(x) = x^2 - 2$

↑ ↓ by 2



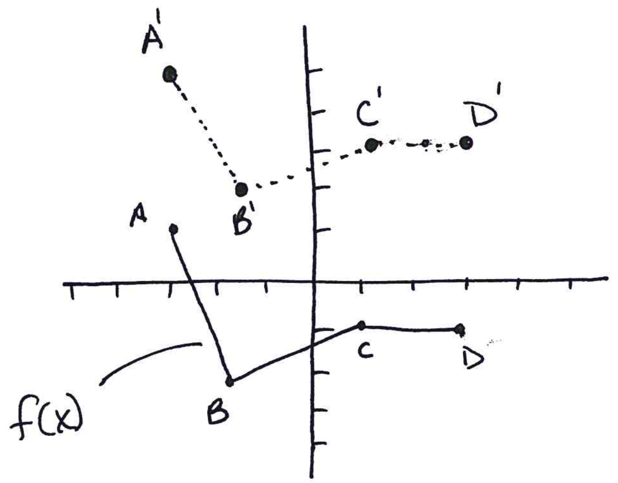
graph $f(x) = x^2$, $g(x) = (x+2)^2$, $h(x) = (x-2)^2$

⇐ ⇒ by 2



Graph Translations by Using Points

When given a graph of the function f(x) you can determine the graph of transformed function by looking at points.



Given the graph f(x) sketch the translation $g(x) = f(x) + 4$

$g(x) = f(x) + 4$

since $y = f(x)$
$g(x) = y + 4$

this is a translation of 4 units up. Only the y value changes

$f(x): (x, f(x))$	$g(x): (x, f(x)+4)$
$A(-3, 1)$	$A'(-3, 5)$
$B(-2, -2)$	$B'(-2, 2)$
$C(1, -1)$	$C'(1, 3)$
$D(3, -1)$	$D'(3, 3)$

Describing Transformations

describe the transformations from base function and graph

$g(x) = (x+5)^2 + 1$

the base function is $f(x) = x^2$

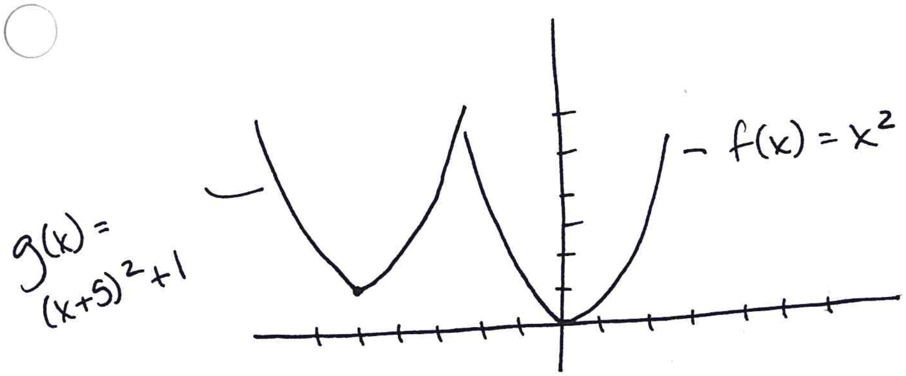
so $g(x) = (x+5)^2 + 1$

$= f(x+5) + 1$

so horizontal translation of -5 5 units left

vertical translation of 1 1 unit up

$f(x) = x^2$



begin at $(0,0)$

move left 5 units this is x $(-5,0)$

move up 1 unit this is y $(-5,1)$

	$f(x) = x^2$	$g(x) = (x+5)^2 + 1$
Domain:	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$
Range:	$\{y \in \mathbb{R}, y \geq 0\}$	$\{y \in \mathbb{R}, y \geq 1\}$

$f(x) = x^2$

$g(x) = (x+5)^2 + 1$ 5 units left

$= f(x+5) + 1$ 1 unit up

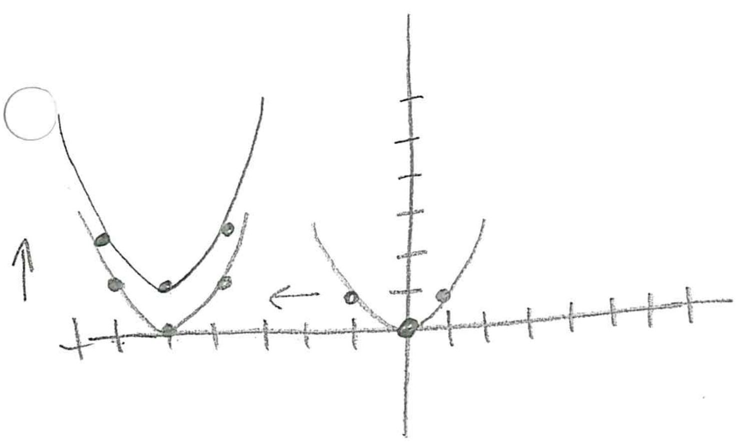
x	(y) x^2
-1	1
0	0
1	1

x	y (x+5)
-1-5=-6	1
0-5=-5	0
1-5=-4	1

chang x

x	(y) (x+5) +1
-6	2
-5	1
-4	2

chang y



5 units over

1 unit up

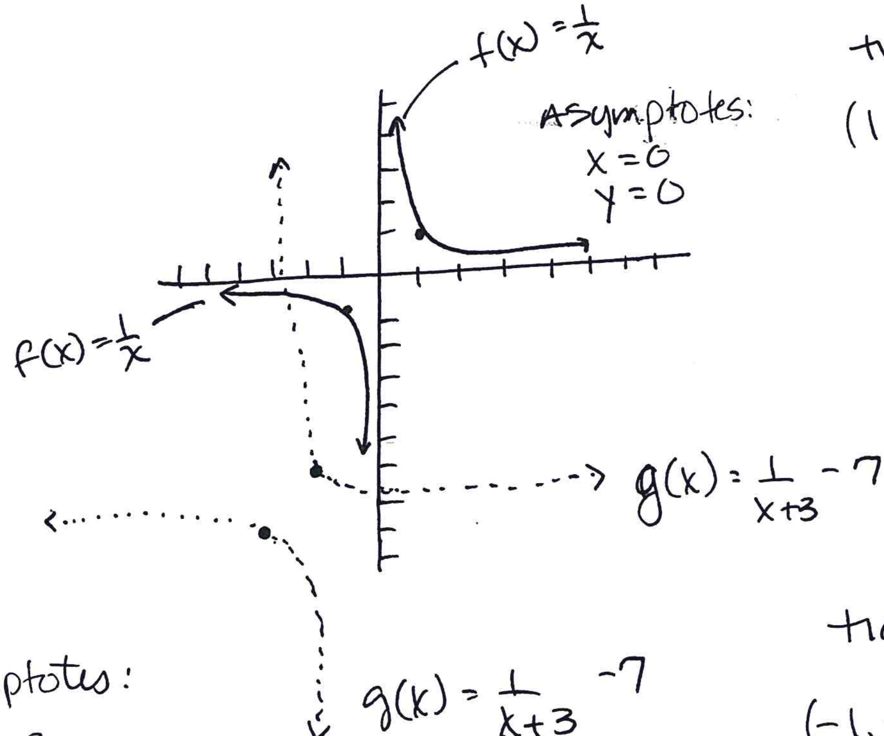
$g(x) = \frac{1}{x+3} - 7$

base function is $f(x) = \frac{1}{x}$

$g(x) = f(x+3) - 7$

horizontal shift 3 units to the left $d = -3$

vertical shift 7 units down $c = -7$



translate from point (1,1)

(1, 1) 3 units to the left (x only)

(−2, 1) then 7 units down (y only)

(−2, −6) new point

translate from point (−1, −1)

$(-1,-1) \xrightarrow[\text{-3 units}]{} (-4,-1)$

$(-4,-1) \xrightarrow[\text{-7 units}]{} (-4,-8)$

asymptotes:
$x = -3$
$x = -7$

$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$
range $\{y \in \mathbb{R}, y \neq 0\}$

$g(x)$: domain $\{x \in \mathbb{R}, x \neq -3\}$
range $\{y \in \mathbb{R}, y \neq -7\}$

the domain & range can be determined by adding the d & c values to restrictions on domain (d) and range (c).

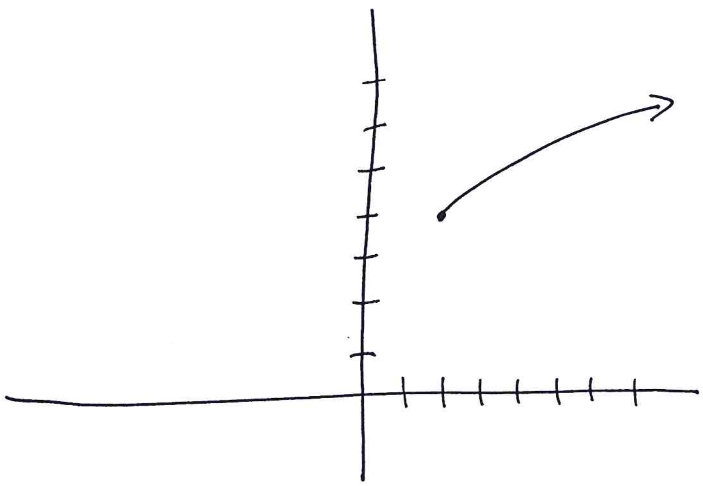
for domain and range

$g(x) = f(x-d) + c$

add the d value to restrictions on domain.
add the c value to restrictions on range.

$g(x) = \sqrt{x-2} + 4$

$d = 2 \quad c = 4$



domain: $\{x \in \mathbb{R},\ x \geq 2\}$
Range: $\{y \in \mathbb{R},\ y \geq 4\}$

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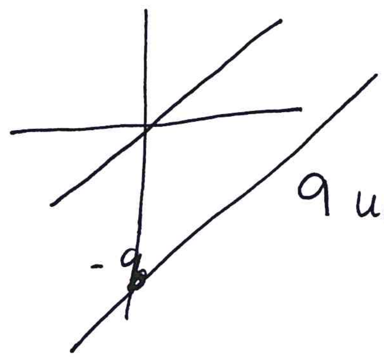
2a $f(x)+5$

A' $(-4, 7)$
B' $(-2, 7)$
C' $(-1, 3)$
D' $(+1, 3)$
E' $(2, 4)$
F' $(4, 4)$

g) $\sqrt{x}$ $y = f(x+10)$ Rachel
h) $\frac{1}{x}$ $y = f(x-2)$ Abi

6 a $f(x)$

$g(x) = f(x) - 9$



9 units down

HW
p 102
1a, b

$\frac{3}{4}$ to ~~better~~ mark

up to 80%

(1a)

x	$f(x)=\sqrt{x}$	$r(x)=f(x)+7$	$s(x)=f(x-1)$
0	0	$0+7=7$	$\sqrt{0-1}$ = no answer
1	1	$1+7=8$	$\sqrt{1-1}=0$
4	2	$2+7=9$	$\sqrt{4-1}=\sqrt{3}$
9	3	$3+7=10$	$\sqrt{9-1}=\sqrt{8}$

$y=\sqrt{x}$

$y=\sqrt{x}+7$

$y=\sqrt{(x-1)}$

(6) a) $f(x)=x$ $\quad y=f(x)-9$

e) $f(x)=x^2$ $\quad y=f(x-6)$

g) $f(x)=\sqrt{x}$ $\quad y=f(x+10)$

h) $f(x)=\frac{1}{x}$ $\quad y=f(x-2)$

$g(x)=x-9$

base function $= f(x)=\cancel{x}$

$y=f(x)-9$

Stretch

a transformation that results in the distance from the x-axis of every point growing by a scale factor greater than 1 (vertical stretch) or the distance from the y-axis of every point growing by a scale factor greater than 1 (horizontal stretch).

Compression

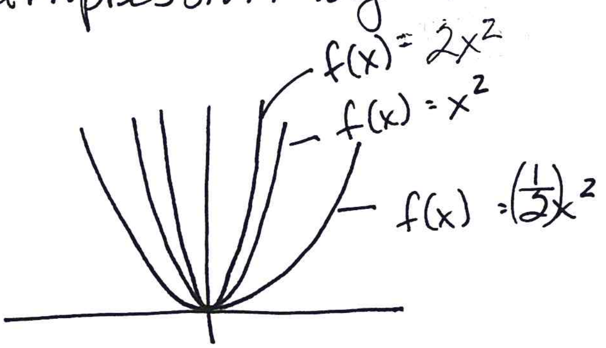
a transformation that results in the distance from the x-axis of every point shrinking by a scale factor between 0 and 1 (vertical compression) or the distance from the y-axis of everypoint shrinking by a scale factor between 0 and 1 (horizontal compression).

2.5 Stretches of functions

The graph of the function $g(x) = af(x)$ where $a > 0$, is a vertical stretch or vertical compression of the graph $f(x)$ by a factor of a.

if $a > 1$, the graph is a vertical stretch by a factor of a.

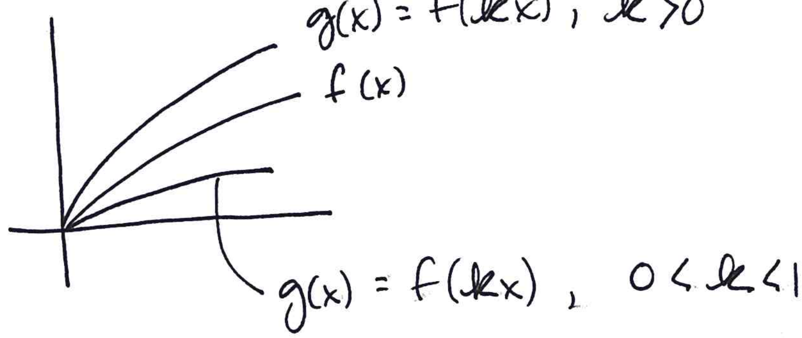
if a is $0 < a < 1$, the graph is a vertical compression by a factor of a.



The graph of the function $g(x) = f(kx)$, $k > 0$, is a horizonal stretch or a horizontal compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$

if $k > 1$, the graph is a horizontal compression by a factor of $\frac{1}{k}$

if k is $0 < k < 1$ the graph is a horizontal stretch by a factor of $\frac{1}{k}$



Graphing Stretches/Compressions

Vertical Stretch

Given the function $f(x) = \sqrt{x}$ write the equation to represent $g(x)$ and graph the transformations

$g(x) = 2f(x)$ $\qquad$ $g(x) = af(x)$

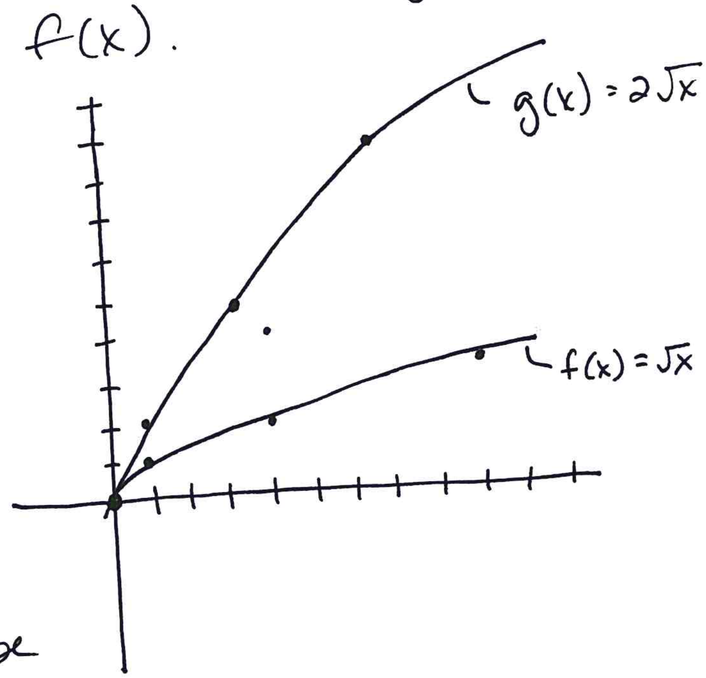
so

$g(x) = 2\sqrt{x}$ $\qquad$ (fill in $f(x)$)

$a = 2$

so the graph is a vertical stretch by a factor of 2 of the graph $f(x)$.

x	(y) $f(x) = \sqrt{x}$	(y) $g(x) = 2\sqrt{x}$
0	0	0
1	1	2
4	2	4
9	3	6



each y-value of $g(x)$ will be twice as far from the x-axis as the corresponding y-value for $f(x)$

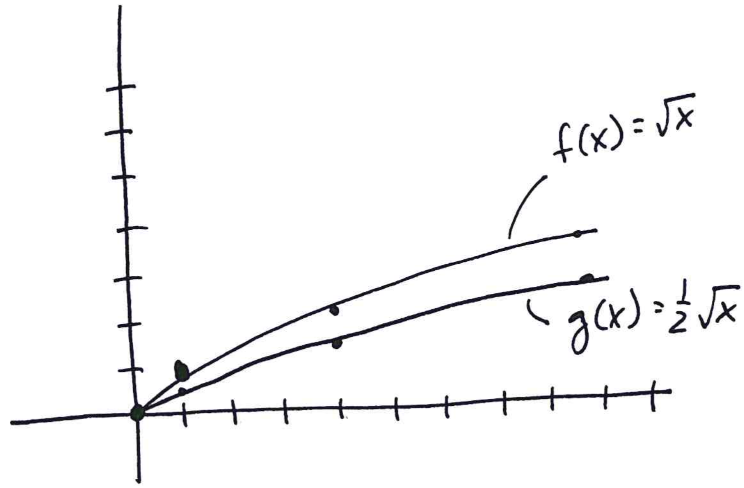
Vertical compression

$g(x) = \frac{1}{2} f(x)$ where $f(x) = \sqrt{x}$

Substitute

$g(x) = \frac{1}{2}\sqrt{x}$

x	$f(x) = \sqrt{x}$ (y)	$g(x) = \frac{1}{2}\sqrt{x}$ (y)
0	0	0
1	1	$\frac{1}{2}$
4	2	1
9	3	$\frac{3}{2}$



each y-value of $g(x)$ will be ½ as far from the x-axis as the corresponding y-value of $f(x)$

Horizontal stretch

$h(x) = f(\frac{1}{2}x)$ $\quad k = \frac{1}{2}$ $\quad \therefore$ horizonal stretch by a factor of $\frac{1}{k}$

or $\frac{1}{\frac{1}{2}} = 2$

given $f(x) = \sqrt{x}$

$h(x) = \sqrt{\frac{1}{2}x}$

Each x-value of $h(x)$ will be twice as far from the y-axis as the corresponding x-value of $f(x)$.

Horizontal compression

$g(x) = f(2x)$

when $f(x) = \sqrt{x}$

$g(x) = \sqrt{2x}$

$k = 2$ this is a horizontal compression by a factor of $\frac{1}{k}$ or $\frac{1}{2}$. The graph will have every x-value of $g(x)$ $\frac{1}{2}$ as far from the y-axis as the corresponding x-values of $f(x)$

x	$f(x) = \sqrt{x}$	$g(x) = \sqrt{2x}$	$h(x) = \sqrt{\frac{1}{2}x}$
0	0	0	0
1	1	$\sqrt{2}$	$\sqrt{1/2}$
2	$\sqrt{2}$	2	1
4	2	$\sqrt{8}$	$\sqrt{2}$
8	$\sqrt{8}$	4	2

$f(x)$

domain $\{x \in \mathbb{R}, \cancel{x} \geq 0\}$

Range $\{y \in \mathbb{R}, y \geq 0\}$

$g(x)$ $h(x)$

same →

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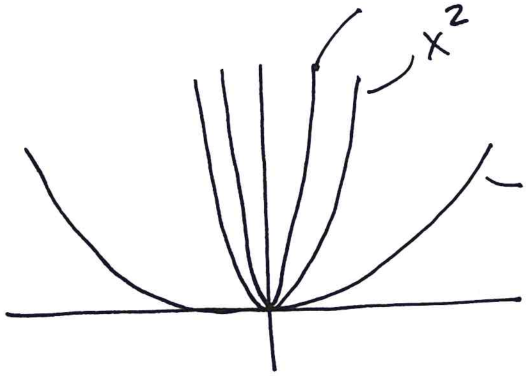
x	$f(x) = x^2$	V. str. $g(x) = 5f(x)$	h. stretch $h(x) = f\left(\frac{1}{4}x\right)$
0	0	0	0
2	4	20	.25 $\left(\frac{1}{4}\right)$ $\left(\frac{1}{4}(2)\right)^2$
4	16	80	1
6	36	180	2.25 $\left(\frac{9}{4}\right)$



$\sqrt{x}$

$f\left(\frac{1}{4}x\right)$

0	0
1	$\sqrt{\frac{1}{4}}$
4	1
9	$\sqrt{\frac{9}{4}}$

Combination of Transformations

Order • Stretches, compressions can be performed in any order before translations.

Make sure • function is written in the form $y = af[k(x-d)] + c$ so you can identify specific transformations.

$a \Rightarrow$ vertical stretch or compression

$k \Rightarrow$ horizontal stretch or compression

$d \Rightarrow$ horizontal translation to the left or right

$c \Rightarrow$ vertical translation up or down

Describe the combination of transformations that are applied to the function f(x) to obtain the transformed function. Write the equation and sketch its graph.

base funtion $f(x) = x^2$
transformed function $g(x) = \frac{1}{2} f(4(x-3)] - 2$

Determine the values of a, k, d & c

$a = \frac{1}{2}$ $k = 4$ $d = 3$ $c = -2$

a is a vertical compression by a factor of 1/2

k is a horizontal compression by a factor of 1/4

d is a horizontal translation 3 units right.

c is a vertical translation 2 units down

Now find the equation from $f(x) = x^2$
$g(x) = \frac{1}{2} f[4(x-3)] - 2$

general form:
$g(x) = af[k(x-d)] + c$

put the transformed equation into the original function

$$g(x) = \frac{1}{2} f\left[4(x-3)\right] - 2$$

$$g(x) = \frac{1}{2}\left(4(x-3)\right)^2 - 2$$

$$= \frac{1}{2}\left(4x-12\right)^2 - 2$$

$$= \frac{1}{2}\left(16x^2 - 96x + 144\right) - 2$$

$$= 8x^2 - 48x + 72 - 2$$

$$= 8x^2 - 48 + 70$$

<u>Sketch the graph</u>

1st do stretch, compression
then translate